FLOW OF A LIQUID-GAS MIXTURE IN A CONTOURED NOZZLE WITH CONSTANT PHASE VELOCITY DIFFERENCE

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A one-dimensional approximation is used to examine simultaneous flow of a gas and of liquid drops in a shaped nozzle under conditions of constant velocity difference between the phases. The nozzle profile to achieve this flow is determined. The parameters of the gas-liquid mixture are obtained in explicit form as a function of the dimensionless gas velocity.

In some situations the need arises to increase the mechanical energy of an incompressible liquid without using equipment involving moving parts and comparatively complex and expensive construction. One method of solving this problem is simultaneous acceleration of a liquid and a gas in liquid-gas nozzles. To do this we are required to calculate the flow of a two-phase gas-liquid medium in these devices. The presence of a considerable amount of liquid in the gas requires calculations of the influence of the liquid phase on the parameters of the carrier gas, a matter which considerably complicates analysis of the flow. Numerical methods [1, 2] are required to solve this problem, even in a one-dimensional formulation, in the general case. In some particular cases it proves possible to find a series of functions describing the nature of the energy transfer between the phases, in explicit form. Such solutions were given in [1] for isobaric and isochoric flow processes of a gas-liquid medium, in which the liquid was distributed in the gas in the form of drops of uniform size.

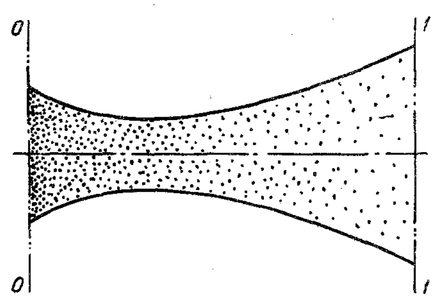

Fig. 1. Schematic of the gas-liquid nozzle.

The present paper examines simultaneous flow of a gas and of liquid drops in a channel of variable cross section, satisfying the condition that the velocity difference of the phases should be constant throughout the whole period of energy transfer between gas and liquid. This condition is of interest in the sense that, for a proper choice of the velocity difference, it is possible to appreciably reduce the dissipation of mechanical energy in the carrier gas in comparison with losses occurring in jet-type equipment of the injector type.

The schematic for computation of the gas-liquid nozzle is shown in Fig. 1.

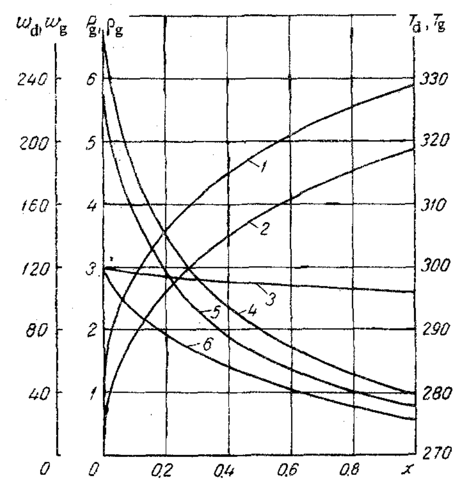

Fig. 2. Distribution of parameters of the gas-liquid mixture along the active part of the nozzle: 1) w_g, m/sec; 2) w_d, m/sec; 3) T_d, °K; 4) ρ_g, kg/m^3; 5) $p_g \cdot 10^{-5}$, N/m^2; 6) T_g, °K; x) axial coordinate, m.

At the section 0–0 the parameters of the gas and liquid and their thermal physical properties are assumed known. Dynamic interaction of the gas and liquid begins at this section and ends at the section 1–1. Sections 0–0 and 1–1 impose a restriction on the contoured part of the nozzle, in which acceleration of the liquid phase is accomplished. The nozzle profile is determined by the ambient parameters of the gas-liquid mixture, describing the assumed conditions of the flow.

The following assumptions are made in the analysis of the motion of the gas and liquid:

1. The problem is solved in a one-dimensional formulation.

2. The mass transfer, chemical reactions between the interacting phases, and the fluxes of enthalpy and momentum through the side walls of the channel are all zero.

3. The liquid phase is distributed in the gas in the form of drops of uniform and identical size. The drops do not interact with one another and with the channel walls.

4. The thermophysical coefficients of the gas and liquid are independent of the mixture parameters.

5. The only force changing the drop momentum is the force of frontal resistance.

The following equations are used to describe the flow of the gas-liquid mixture in the nozzle:

Equations of motion for the gas and drops:

$$m_g dw_g = -f_g dP - m_d dw_d, \tag{1}$$

$$M_d dw_d = C_x \rho_g \frac{V^2}{2} S_d \, d\tau. \tag{2}$$

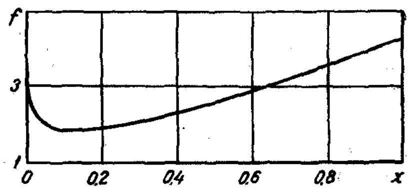

Fig. 3. Profile of the gas-liquid nozzle: $f \cdot 10^3$) transverse cross section of the nozzle, m^2; x) axial coordinate, m.

The energy equation:

$$c_p dT_g + w_g dw_g = -uc_d dT_d - uw_d dw_d. \tag{3}$$

The continuity equations for the gas and liquid:

$$m_g = \rho_g w_g f_g, \tag{4}$$

$$m_d = \rho_d w_d f_d. \tag{5}$$

The heat transfer equation:

$$c_d \, dT_d = \alpha \frac{F_d}{M_d} (T_g - T_d) \, d\tau. \tag{6}$$

The equation of state for the gas:

$$P = \rho_g R T_g. \tag{7}$$

The condition for constant difference of velocity between the phases:

$$V = w_g - w_d = \text{const}. \tag{8}$$

We shall calculate the change in resistance coefficient of a liquid drop and in the coefficient of heat transfer from the gas to the drop by means of the following relations [3, 4]:

$$C_x = a_1/\sqrt{\mathrm{Re}}, \tag{9}$$

$$\alpha = \frac{\lambda_g}{d_d} a_2 \sqrt{\mathrm{Re}}. \tag{10}$$

A boundary condition for the system of equations (1)–(10) is the given parameters of the gas-liquid mixture at the initial nozzle section ($P_0, T_{g0}, \rho_{g0}, T_{d0}, w_{d0}, w_{g0}, f_{g0}, f_{d0}, C_{x0}, \alpha_0, \tau_0 = 0$).

By solving the system (1)–(10), the desired functions are found in the following form:

The temperature of the liquid drops

$$T_d = A_{d1}\omega^2 + A_{d2}\omega + A_{d3} + A_{d4}\exp(-N\omega), \tag{11}$$

where

$$A_{d1} = -\frac{w_{g0}^2}{2c_p}\frac{u+1}{\beta+1},$$

$$A_{d2} = \frac{w_{g0}^2}{c_p}\left[\frac{u+1}{A(\beta+1)^2} - \frac{1+u-uV/w_{g0}}{\beta+1}\right],$$

$$A_{d3} = \frac{B}{c_p(\beta+1)} + \frac{w_{g0}^2(1+u-uV/w_{g0})}{c_p A(\beta+1)^2} - \frac{w_{g0}(u+1)}{c_p A^2(\beta+1)^3},$$

$$A_{d4} = T_{d0} - A_{d3}, \quad \beta = u\frac{c_d}{c_p}, \quad N = A(\beta+1),$$

$$A = 8\frac{a_2}{a_1}\frac{\lambda_g w_{c0}}{\mu_g c_d V}, \quad B = c_p T_{g0} + uc_d T_{d0}. \tag{12}$$

Here, and in what follows, we use the dimensionless gas velocity $\omega = w_g/w_{g0} - 1$ as the independent variable. At the initial section $\omega = 0$.

The gas temperature is

$$T_g = A_{g1}\omega^2 + A_{g2}\omega + A_{g3} + A_{g4}\exp(-N\omega), \tag{13}$$

where

$$A_{g1} = -\beta A_{d1} - \frac{w_{g0}^2}{2c_p}(u+1),$$

$$A_{g2} = -\beta A_{d2} - (1+u-uV/w_{g0})\frac{w_{g0}^2}{c_p},$$

$$A_{g3} = \frac{B}{c_p} - \beta A_{d3}, \quad A_{g4} = -\beta A_{d4}. \tag{14}$$

The gas pressure is

$$\frac{P_0}{P} = \exp\left[\frac{(u+1)c_p}{R} \times \int_0^{\omega} \frac{(\omega+1)\,d\omega}{A_1\omega^2 + A_2\omega + A_3 + A_4\exp(-N\omega)}\right], \tag{15}$$

where

$$A_i = A_{gi}\frac{c_p}{w_{g0}^2} \quad (i = 1, 2, 3, 4).$$

The integral in (15) is not expressed in terms of elementary functions, but when heat transfer between the phases can be neglected, a relation $P = P(\omega)$ can be obtained in the form

$$\frac{P_0}{P} = \left(\frac{A_1\omega^2 + A_2\omega + A_3}{A_3}\right)^{\frac{(u+1)c_p}{2RA_1}} \times \left[\frac{(\varepsilon - 2A_1\omega - A_2)(\varepsilon + A_2)}{(\varepsilon + 2A_1\omega + A_2)(\varepsilon - A_2)}\right]^{\frac{c_p(u+1)}{R\varepsilon}\left(1 - \frac{A_2}{2A_1}\right)}, \tag{16}$$

where

$$\varepsilon = \sqrt{A_2^2 - 4A_1A_3}.$$

The geometric parameters of the nozzle (current length and area of active cross section of the gas and liquid phases)

$$x = \frac{1}{A_x}\int_0^{\omega} \frac{\omega\,d\omega}{\rho_g^{0.5}(\omega)}, \tag{17}$$

where

$$A_x = \frac{3a_1 \mu_g^{0.5} V^{1.5}}{4d_d^{1.5} \rho_d w_{g0}^2},$$

$$\rho_g(\omega) = \frac{P(\omega)}{RT_g(\omega)};$$

$$f_g = \frac{m_c}{\rho_g(\omega)(1+\omega) w_{g0}}; \quad (18)$$

$$f_d = \frac{m_d}{\rho_d (w_{d0} + w_{g0}\omega)}. \quad (19)$$

The velocity of the gas and of the liquid drops

$$w_g = w_{g0}(1+\omega),$$
$$w_d = w_{d0} + w_{g0}\omega. \quad (20)$$

The data obtained allow calculation of the remaining parameters of interest of the gas-liquid medium (entropy of the gas phase, energy transmitted to the liquid, etc.).

By way of example, Figure 2 gives graphs showing the change of the parameters of the gas-liquid flow along the active part of the nozzle for one case of flow of an air-water mixture. The parameters of the air and water at the initial section were assumed as follows: $P_0 = 5.88 \cdot 10^5$ N/m^2, $T_{g0} = 300°$ K, u = 5, m_g = = 1 kg/sec, V = 40 m/sec, $\rho_{g0} = 6.84$ kg/m^3, $w_{d0} = 0$, $T_{d0} = 300°$ K, $d_d = 0.932 \cdot 10^{-4}$ m. At the nozzle exit the gas pressure was taken to be $P_1 = 0.98 \cdot 10^5$ N/m^2.

Figure 3 shows the nozzle profile obtained by calculation. It should be noted that, in spite of the flow regime of the gas being subsonic, a convergent-divergent nozzle profile is obtained. It is also characteristic that the gas velocity at the nozzle throat is 125 m/sec, which is considerably less than the velocity of sound in air for the appropriate temperature.

NOTATION

m_g, w_g, ρ_g, T_g, c_p, P, μ_g, λ_g, and R are the mass flow, velocity, density, thermodynamic temperature, specific heat at constant pressure, pressure, dynamic viscosity, thermal conductivity, and gas constant for the gas phase; m_d, w_d, ρ_d, T_d, and c_d denote the flow rate, velocity, density, thermodynamic temperature, and specific heat of the liquid phase; f_g, f_d are the transverse flow cross sections assumed for the gas and liquid phases, respectively; M_d, S_d, F_d, and d_d are the mass, mid-section dimension, surface area and diameter of a drop; C_x is the resistance coefficient of a drop; α is the coefficient of heat transfer between the gas and the drop; τ is time; x is the distance along the nozzle axis, reckoned from the initial section; a_1 = 10.5 to 14, a_2 = 0.54 are empirical constants [3, 4]; $u = m_d/m_g$; $Re = \rho_g V d_d/\mu_g$.

REFERENCES

1. V. A. Bashkatov and A. A. Tsvetkova, Izv. SO AN SSSR, ser. tekhn. nauk, issue 2, no. 6, 1965.

2. Collection: Basic Gasdynamics, ed. G. Emmons [Russian translation], IL, 1963.

3. A. S. Lyshevskii, Izv. vuzov, Energetika, no. 7, 1963.

4. D. N. Vyrubov, ZhTF, 9, no. 21, 1939.

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